16 Confinement in SUSY QCD: Part I

16.1 $F = N$: Chiral Symmetry Breaking

Recall that the classical moduli space of SUSY QCD is parameterized by

$$
\begin{aligned}
M_i^j &= \overline{\Phi}^{j\alpha}\Phi_{\alpha i} && (16.1)\\
B_{i_1,\ldots,i_N} &= \Phi_{\alpha_1 i_1}\ldots\Phi_{\alpha_N i_N}\epsilon^{\alpha_1,\ldots,\alpha_N} && (16.2)\\
\overline{B}^{i_1,\ldots,i_N} &= \overline{\Phi}^{\alpha_1 i_1}\ldots\overline{\Phi}^{\alpha_N i_N}\epsilon_{\alpha_1,\ldots,\alpha_N} && (16.3)\\
& && (16.4)
\end{aligned}
$$

For $F = N$ the baryons are flavor singlets:

$$
\begin{aligned}
B &= \epsilon^{i_1,\ldots,i_F}B_{i_1,\ldots,i_F} && (16.5)\\
\overline{B} &= \epsilon_{i_1,\ldots,1_F}\overline{B}^{i_1,\ldots,i_F} && (16.6)
\end{aligned}
$$

Clasically these fields satisfy a constraint:

$$\mathrm{det}M = B\overline{B} \tag{16.7}$$

With quark masses turned on we have:

$$\langle M_i^j\rangle = (m^{-1})_i^j\left(\mathrm{det}m\Lambda^{3N-F}\right)^{\frac{1}{N}} \tag{16.8}$$

Taking a determinant (and using $F = N$) we have

$$\mathrm{det}\langle M\rangle = (\mathrm{det})^{-1}\mathrm{det}m\Lambda^{2N} = \Lambda^{2N} \tag{16.9}$$

independent of the masses. However $\mathrm{det}m \neq 0$ sets $\langle B\rangle = \langle\overline{B}\rangle = 0$, since we can integrate out all the fields that have baryon number. Thus the classical constraint is violated! To understand what is going on it is helpful to use holomorphy and the symmetries of the theory. The flavor invariants are:

	$U(1)_A$	$U(1)$	$U(1)_R$
$\mathrm{det}M$	$2N$	0	0
B	N	N	0
$\overline{B}$	N	$-N$	0
Λ^{2N}	$2N$	0	0

Thus the generalized form of the constraint that has the correct $\Lambda \to 0$ and B,$\overline{B} \to 0$ limits is

$$\mathrm{det}M - \overline{B}B = \Lambda^{2N}\left(1 + \sum_{ab} C_{ab}\frac{\Lambda^2 Na(\overline{B}B)^b}{\mathrm{det}M}\right) \tag{16.10}$$

For $\langle \overline{B}B \rangle \gg \Lambda^{2N}$ the theory is perturbative, but with $C_{ab} \neq 0$ we find solutions of the form

$$\det M = \left(\overline{B}B\right)^{\frac{b-1}{a+b}} \tag{16.11}$$

which do not reproduce the perturbative limit, so we conclude $C_{ab} = 0$. So the quantum constraint is:

$$\det M - \overline{B}B = \Lambda^{2N} \tag{16.12}$$

First note that this equation has the correct form to be an instanton effect. Also note that we cannot take $M = B = \overline{B} = 0$ (this is referred to as having a "deformed" moduli space). This means that the (global) gauge symmetries are (partially) broken everywhere in the quantum moduli space. For example consider the following special points with enhanced symmetry: when $M_i^j = \Lambda^2 \delta_i^j$, $B = \overline{B} = 0$ the global symmetry is broken to $SU(F)_d \times U(1) \times U(1)_R$, while at $M = 0$, $B\overline{B} = -\Lambda^{2N}$ the global symmetry is broken to $SU(F) \times SU(F) \times U(1)_R$. For large VEVS we can understand this as squark VEVs giving masses to quarks and gauginos. There is no place in the moduli space where gluons become light, so there are no singular points and the moduli space is smooth. This is an example of a theory that exhibits complementarity since we can go smoothly from a Higgs phase (large VEVS) to a confining phase (VEVS of $\mathcal{O}(\Lambda)$) without going through a phase transition. This is true because the squarks are in the fundamental representation.

We can impliment the constraint in superpotential form by using a Lagrange mulitplier field.

$$W = X\left(\det M - \overline{B}B - \Lambda^{2N}\right) \tag{16.13}$$

Adding a mass for the Nth flavor and writing

$$M = \begin{pmatrix} \widetilde{M}_i^j & N^j \\ P_i & Y \end{pmatrix} \tag{16.14}$$

so $\Delta W = mY$, we have the following equations of motion:

$$\frac{\delta W}{\delta B} = -X\overline{B} \tag{16.15}$$

$$\frac{\delta W}{\delta \overline{B}} = -XB \tag{16.16}$$

$$\frac{\delta W}{\delta N^j} = X \mathrm{cofac}(N^j) \tag{16.17}$$
$$\frac{\delta W}{\delta P_i} = X \mathrm{cofac}(P_i) \tag{16.18}$$
$$\frac{\delta W}{\delta Y} = X \det \widetilde{M} + m \tag{16.19}$$
$$\tag{16.20}$$

So

$$X = m \left(\det \widetilde{M}\right)^{-1} \tag{16.21}$$
$$B = \overline{B} = N^j = P_i = 0 \tag{16.22}$$

Also

$$\frac{\delta W}{\delta X} = Y \det \widetilde{M} - \Lambda^{2N} \tag{16.23}$$

So the effective superpotential is

$$\begin{aligned} W_{\mathrm{eff}} &= \frac{m\Lambda^{2N}}{\det \widetilde{M}} \\ &= \frac{\Lambda^{2N+1}_{N,N-1}}{\det \widetilde{M}} \end{aligned} \tag{16.24}$$

Which is just the ADS superpotential, as we would expect for $SU(N)$ with $N-1$ flavors

Let's consider in more detail the points in the moduli space with enhanced symmetry. When $M^j_i = \Lambda^2 \delta^j_i$, $B = \overline{B} = 0$ the global symmetry is broken to $SU(F)_d \times U(1) \times U(1)_R$. In terms of the elementary fields, the Φ and $\overline{\Phi}$ VEV's break $SU(N) \times SU(F) \times SU(F)$ to $SU(F)_d$. The quarks transform as $\mathbf{1} + \mathbf{Ad}$ under this diagonal group, while the gluino transforms as $\mathbf{Ad}$. The composites transform as:

	$SU(F)_d$	$U(1)$	$U(1)_R$
$M - \mathrm{Tr}M$	$\mathbf{Ad}$	0	0
$\mathrm{Tr}M$	$\mathbf{1}$	0	0
B	$\mathbf{1}$	N	0
$\overline{B}$	N	$-N$	0

The field $\mathrm{Tr}M$ gets a mass with the Lagrange multiplier field.

The anomalies match as follows (recalling that $F = N$):

$$\begin{array}{lcc} & \text{elem.} & \text{comp.} \\ U(1)^2 U(1)_R : & -2FN & -2N^2 \\ U(1)_R : & -2FN + N^2 - 1 & -(F^2-1) - 1 - 1 \\ U(1)_R^3 : & -2FN + N^2 - 1 & -(F^2-1) - 1 - 1 \\ U(1)_R SU(F)_d^2 : & -2N + N & -N \end{array} \tag{16.25}$$

At $M = 0$, $B\overline{B} = -\Lambda^{2N}$ only the $U(1)$ is broken. A linear combination of B and $\overline{B}$ gets a mass with the Lagrange mulitplier field. The anomalies match as follows:

$$\begin{array}{lcc} & \text{elem.} & \text{comp.} \\ SU(F)^3 : & N & F \\ U(1)_R SU(F)^2 : & -N\frac{1}{2} & -F\frac{1}{2} \\ U(1)_R : & -2FN + N^2 - 1 & -F^2 - 1 \\ U(1)_R^3 : & -2FN + N^2 - 1 & -(F^2-1) - 1 - 1 \end{array} \tag{16.26}$$

which agree because $F = N$.

References

[1] "Lectures on supersymmetric gauge theories and electric-magnetic duality," by K. Intriligator and N. Seiberg, hep-th/9509066.